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UNDERLYING METHODOLOGY OF THE MODELS IN OFS MARKET RISK

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Introduction to Underlying Methodology of the Models in OFS Market Risk

This is a technical whitepaper which explains how Value at Risk (VaR) is computed using Volatility Estimation and VaR Estimation models.

1. Volatility and Correlation Estimation

The Volatility and Correlation process involves estimating the Variance, Covariance and Correlations between the historical returns of following risk factors:

- Interest Rate
- Equity
- Commodities
- Exchange Rate

This process is calculated based on the following common, EWMA and GARCH specific parameters:

- **Missing Price Estimation Method:** Estimation of missing prices helps to ensure the accuracy of data and computations made resembles the market conditions. It also suffices the positive semi-definiteness of the covariance matrix which ensures to compute realistic VaR. You can specify the method for estimation of missing prices. The supported estimation methods are Linear Interpolation, nearest, prior and omit functions.
- **Number of Observation Days:** You can specify the number of days to be considered for estimation of Variance-Covariance.
- **Seasonality Parameter Specification:** You can specify multiple seasons and select only those historical observations occurring in the season that the current date falls under for the purpose of estimating volatilities and correlations. For instance, oil prices are high during the winter months as the demand for oil is higher during this period. In this case, calculating volatilities using the entire year's observations provides an incorrect estimate. Suppose seasonality is specified as 3 seasons as per which the winter season starts from 1st December and ends on 31st Marc. In this case, if the MIS date falls within this period, only those historical observations occurring during the months of December, January, February and March is considered for estimation of volatilities and correlations. This provides a more precise estimate of the volatilities.
- **Cross Correlations:** You can also specify if correlations between the various asset classes are to be considered or to be taken as zero. This does not affect the intra-asset class correlations. For instance, for a given portfolio consisting of US equities, US interest rate instruments and Indian equities, if cross correlations is specified as False then the correlation between Indian equities and US interest rates is changed to zero while the correlation between US equities and Indian equities remains unchanged.
- **Decay Factor (Lambda):** Decay factor specifies the weight to be given to the various observation days. A lambda of 1 indicates equal weights given to all the

observations. A value closer to zero gives more weight age to the recent observation days.

- **Calibrating GARCH Model:** You can choose to recalibrate the GARCH model or re-use the pre-calibrated values. In case of the first execution of this model, the interim model parameters need to be calibrated from historical observations. Subsequently, they can be re-used for estimating volatilities.

1.1 Exponentially Weighted Moving Averages (EWMA) Method

The risk factors are correlated and risk estimation models are factored in the calculations. OFS Market Risk estimates the Variance of, Covariance and correlations between the historical returns of risk factors using the Exponentially Weighted Moving Averages (EWMA) Method.

EWMA method allows for greater weight being given to recent observations. These risk factors are values of interest rates, exchange rates, equities and commodities at the relevant Vertices.

The number of observations to be used for the Variance-Covariance estimation is specified as part of the Market Risk VaR Model.

Volatilities and correlations of certain asset classes, such as commodities, may be heavily influenced by seasons. OFS Market Risk allows adjustments to calculations to factor in the effect of seasons. Seasonality adjustment requires specification of season start and end dates. When seasonality is specified, only those historical returns on risk factors for the observations that fall within the relevant season are used for computations.

For example, if the number of seasons specified is 3, then 3 dates are to be provided that is, first date for start of season 1, second date for end of Season 1 and third date for end of season 2).

Where seasonality is specified the price observations are taken from seasons in which the valuation date falls.

Returns Calculation

The first step in estimation of variance-covariance is the calculation of returns for each risk factor. Returns are calculated over the historical time period in intervals corresponding to the VaR horizon. Example, for an observation period of 250 days and a VaR horizon of 2-days, a total of 124 non-overlapping returns are calculated. The return over a time interval is calculated using the following formula:

$$R_t = \ln(P_t) - \ln(P_{t-n})$$

Where,

$$P_t = \text{Price at time } t$$

$$P_{t-n} = \text{Price at time } t - n.$$

Estimation of Variance and Volatility

Variance of each of the risk factor's returns over the VaR Horizon is calculated using the following formula:

$$\sigma R_t = \frac{\sum_{t=1}^T (\lambda^{T-t} * R_t^2)}{\sum_{t=1}^T (\lambda^{T-t})}$$

Where,

σR_t = Estimated volatility of risk factor R_t

R_t = Return on risk factors at time t

T = Number of historical observations

t = Day on which a particular historical observation has occurred

λ^{T-t} = Decay factor which gives the relative weight $(T - t)$ to each observation

Volatility is calculated using the following formula:

$$\text{Volatility} = \sqrt{\sigma R_t}$$

Estimation of Covariance

Covariance is a measure of the extent of joint movement in two variables which in case of market risk are the risk factors. OFS Market Risk estimates the covariance between each pair of risk factors as per the formula given:

$$\text{Cov} (R_{t1}, R_{t2}) = \frac{\sum_{t=1}^T (\lambda^{T-t} R_{t1} R_{t2})}{\sum_{t=1}^T \lambda^{T-t}}$$

Where,

$\text{Cov} (R_{t1}, R_{t2})$ = Covariance between risk factor returns R_{t1} and R_{t2}

R_{ti} = Return on at time "t" on risk factor i

Estimation of Correlation

Correlation is a measure of the strength of the joint movement in two variables. In case of market risk these are the risk factors. OFS Market Risk estimates the correlation between each pair of risk factors as per the formula given:

$$\rho(R_{t1}, R_{t2}) = \frac{\text{Cov} (R_{t1}, R_{t2})}{\sigma R_{t1} \sigma R_{t2}}$$

Where,

$$\rho(R_{t1}, R_{t2}) = \text{Correlation between risk factor returns } R_{t1} \text{ and } R_{t2}$$
$$\sigma R_t = \text{Estimated volatility of risk factor } R_t$$
$$\text{Cov}(R_{t1}, R_{t2}) = \text{Covariance between risk factor returns } R_{t1} \text{ and } R_{t2}$$
Cross Correlations

Where cross correlations are not considered in a given run, the value of correlation across Asset Class is reset as 0. Intra Asset Class correlations are unchanged. Example, Correlation between interest rates of different maturities and across currencies is considered while correlation between Interest Rate Asset Classes and Stock Indices is reset to 0. Whether cross-correlations are to be considered or not is specified as part of Market Risk VaR Model definition.

1.2 Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Method

The volatilities and correlations are estimated using both the EWMA methods as well as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) method. The volatility – correlation calculations take into account the seasonality parameters, cross correlations and the horizon specified.

Estimation of GARCH Parameters

OFS Market Risk utilizes the Linear Programming Problem function of NAG libraries for the estimation of the GARCH parameters. Alpha (α), Beta (β) and Omega (ω). α , β and ω are estimated by maximizing following equation:

$$mlecoeff = \sum_m^{t=1} \left[-\ln(Cov_t) - \left(\frac{X_{t-1}Y_{t-1}}{Cov_t} \right) \right]$$

Where,

$m =$ number of observation days

$X_{t-1} =$ Returns of risk factor X on day $t - 1$

$Y_{t-1} =$ Returns of risk factor Y on day $t - 1$

$Cov_t = \omega + \alpha(X_{t-1}Y_{t-1}) + \beta CovXY_{t-1}$

$CovXY_{t-1} =$ Covariance between risk factor X and Y on day $t - 1$

α, β and $\omega =$ GARCH Parameters

$\omega = V_L (1 - \alpha - \beta)$

$V_L =$ Long term covariance = $\frac{\sum_m^{t=1} (X_{t-1} - \mu_X)(Y_{t-1} - \mu_Y)}{m}$

$\mu_X =$ Mean of Risk factor X on time $t - 1$

$\mu_Y =$ Mean of Risk factor Y on time $t - 1$

With constraints,

$$\alpha, \beta, \omega > 0$$

$$\alpha + \beta < 1$$

The variance is predicted for each day using the guess values for α , β and ω . The maximum likelihood estimate for each day is computed using the formula given above. Values of α , β and ω are estimated using the LPP function of NAG libraries in an iterative manner such that the overall MLE during the observation period is maximized.

Following are the detailed steps:

1. Generate guess parameter of α , β and ω such that $\alpha, \beta, \omega > 0$ and $\alpha + \beta < 1$
2. Calculate mlecoeff for scenarios
3. Repeat step 1 and 2 for 3000 scenarios.
4. Select the scenario where mlecoeff is maximum and use the corresponding α , β and ω values to derive variance/covariance and correlation using equations explained in section below.

Alternatively, GARCH parameters can be given as download and while defining model user can select option of not to calibrate the GARCH parameters.

Estimation of Variance and Volatility

The formula for estimating variance using GARCH model is as follows:

$$\text{Var}X_t = \omega + \alpha(X_{t-1}X_{t-1}) + \beta \text{Var}X_{t-1}$$

Where,

$$\text{Var}X_t = \text{Variance of risk factor } X \text{ on day } t$$

$$\text{Var}X_{t-1} = \text{Variance of risk factor } X \text{ on day } t - 1$$

$$X_{t-1} = \text{Returns of risk factor } X \text{ on day } t - 1$$

$$\alpha, \beta \text{ and } \omega = \text{GARCH Parameters estimated in above step}$$

On the first day when the GARCH model is applied, the predicted variance as on the previous day is not available. In this case, the first value of predicted variance is taken as the square of the returns. Thereafter, the GARCH model is applied to predict the variance. Volatility is calculated using the following formula:

$$\sigma X_t = \sqrt{\text{Var}X_t}$$

Where,

$$\sigma X_t = \text{Volatility of Risk Factor } X \text{ on day } t$$

$$\text{Var}X_t = \text{Variance of risk factor } X \text{ on day } t$$

Covariance based on the GARCH model is estimated using the formula as follows:

$$Cov_t = \omega + \alpha(X_{t-1}Y_{t-1}) + \beta CovXY_{t-1}$$

Where,

X_{t-1} = Returns of risk factor X on day t – 1

Y_{t-1} = Returns of risk factor Y on day t – 1

$CovXY_{t-1}$ = Covariance between risk factor X and Y on day t – 1

On the first day when the GARCH model is applied, the predicted covariance as on the previous day is not available. In this case, the first value of predicted covariance is taken as the square of the returns. Thereafter, the GARCH model is applied to predict the covariance.

Estimation of Correlation

Correlation is a measure of the strength of the joint movement in two variables. In case of market risk these are the risk factors. OFS Market Risk estimates the correlation between each pair of risk factors as per the formula given:

$$\rho_{XY_t} = \frac{CovXY_t}{\sigma_{X_t}\sigma_{Y_t}}$$

Where,

ρ_{XY_t} = Correlation between risk factor X and Y on day t

$CovXY_t$ = Covariance between risk factor X and Y on day t

σ_{X_t} = Volatility of Risk Factor X on day t

σ_{Y_t} = Volatility of Risk Factor Y on day t

Cross Correlations

Where cross correlations are not considered in a given run, the value of correlation across Asset Class is reset as 0. Intra Asset Class correlations are unchanged. Example, Correlation between interest rates of different maturities and across currencies is considered while correlation between Interest Rate Asset Classes and Stock Indices is reset to 0. Whether cross-correlations are to be considered or not is specified as part of Market Risk VaR Model definition.

2. VaR Estimation

Value-at-Risk (VaR) is the worst expected portfolio loss under normal market conditions over a specific time interval at a given confidence level.

2.1 Analytic (Parametric / Variance Covariance Method) Method

Calculation of VaR of a portfolio is a two-step process as follows:

- **Calculation of Undiversified VaR:** This method is calculated by multiplying the volatility vector with the cash flows and summing them up. The output of this step is the undiversified VaR corresponding to each Asset – Asset Class – Maturity vertex as well as the undiversified VaR of the portfolio.
- **Calculation of VaR: Diversified VaR:** This method is computed by multiplying the vector of undiversified VaR with the correlation matrix. The output of this step is the diversified portfolio VaR in the reporting currency.

Calculation of Undiversified VaR

The first step in estimating the portfolio VaR is to estimate the undiversified VaR of each of the positions. The undiversified VaR of each Asset – Asset Class – Maturity vertex is arrived at by multiplying the cash flow matrix with the respective risk factor volatilities. The cash flow matrix is an $n \times 1$ matrix which has rows equal to the number of Asset - Asset Class – Maturity combination and 1 column holding cashflows.

$$VU = z\alpha \left(\frac{V}{\sqrt{360}} \right)$$

Where,

$VU =$ Vector of Undiversified VaR for each position.

$CF =$ $n \times 1$ Vector of Cash Flows

$V =$ Annual Volatility Vector

$z\alpha =$ Standard normal value, z , corresponding to the confidence level α

The Undiversified VaR of the Portfolio is the simple summation of the elements of the undiversified VaR vector.

Calculation of (Diversified) VaR

OFS Market Risk estimates the VaR of a portfolio as function of Cashflow Vector and the Variance Covariance matrix. . Here, the VaR is calculated at the portfolio level.

$$VaRp = (CFT * Q * CF)^{1/2}$$

Where,

$$VaRP = Portfolio VaR$$

$$CF = Cashflow Matrix$$

$$CFT = Transpose of the vector of Cashflow Matrix$$

$$Q = VarianceCovariance Matrix * z\alpha^2$$

$$z\alpha = Standard normal value, z, corresponding to the confidence level \alpha$$

Component VaR

Under the Analytic Method to risk measure estimation, Component VaR is calculated as the contribution of each Asset – Asset Class – Maturity vertex to the Diversified Portfolio VaR as follows:

- Delta VaR of each Asset – Asset Class – Maturity vertex maturity is calculated as follows:

$$DelVaR = \frac{(CFT * Q)}{VaRP}$$

Where,

$$DelVaR = Delta VaR of each Asset, Asset Class and Maturity vertex$$

$$CFT = Transpose of the Cash Flow Vector$$

$$z\alpha^2 = Square of standard normal value, z, at a confidence level of \alpha$$

$$VaRP = Diversified Portfolio VaR$$

$$Q = VarianceCovariance Matrix * z\alpha^2$$

- Component VaR is calculated as follows:

$$Component VaR = Cash Flow * the respective DelVaR$$

2.2 Historical Simulation Method

Simulation of Risk Factor

Simulation is carried out by applying historical returns on the current value of risk factors based on the parameters specified in the Market Risk VaR Model.

The output of this process is the simulated values of all risk factors in a given portfolio. The number of simulated values of each risk factor is equal to n-1 number of historical observations specified in the Market Risk VaR Model.

For historical simulation methods, returns on risk factors are calculated using the returns method. The returns observed historically are applied to the current value of the risk factors to simulate the

possible values of the risk factors. The returns are calculated over the historical time period in intervals corresponding to the VaR Horizon.

Stock index is the risk factor for equity instruments. The returns are thus calculated on the historical values of stock index and stocks specified as custom equity risk factors. The equity prices are calculated from the stock index returns.

Simulation of Portfolio Value

Simulated portfolio value refers to the price of portfolio under each scenario.

The instrument value for all instrument as part of given portfolio is calculated under all simulated scenario. Simulated portfolio value in the reporting currency is derived as sum of all instrument prices under each scenario.

P&L Distribution

P&L Distribution is Profit or Loss of the portfolio under each scenario against the portfolio value as on the current day.

Portfolio VaR and CVaR

The VaR and CVaR values which are obtained from the P&L Distribution are based on the confidence level and horizon specified in the Market Risk VaR Model.

Based on the confidence level “ α ” the 1-day α -VaR is the $(1 - \alpha)$ th value in the arranged P&L values. The confidence level “ α ” must be entered in the OFS Market Risk application. The t-day α -VaR is estimated by multiplying the 1-day α -VaR by \sqrt{t} .

CVaR is calculated as the expected value of all simulated data points that exceed the VaR. It is the expected loss conditional on the occurrence of loss greater than VaR. The P&L distribution and is calculated as:

$$\text{CVaR} = E(Y / Y > \text{VaR})$$

Where,

Y : Value of Simulated Loss

Thus, it is the arithmetic average of all simulation points exceeding the VaR value.

Component VaR

Under the Simulation Methods to risk measure estimation, both Monte Carlo as well as Historical, Component VaR is calculated as the contribution of each position to the Portfolio VaR as follows:

- The scenario position values are estimated for “n” instruments in the portfolio for “m” scenarios of prices and converted to the home currency.
- The variance of the scenario portfolio values is estimated as follows:

$$\text{Variance} = 1/m \sum_{j=1}^m (jP_{t+1} - \mu_j)^2$$

Where,

- m : Number of scenario portfolio values
 j_{Pt+1} : Scenario portfolio value
 μ_j : Mean of the scenario portfolio values

- The covariance between the scenario position values of each instrument and the scenario portfolio values is estimated as follows:

$$\text{Covariance} = 1/m \sum_{k=1}^m \sum_{j=1}^m (x_k - \mu_k) (j_{Pt+1} - \mu_j)$$

Where,

- x_k : Scenario position values of instrument “i”

The covariance of each position with the scenario portfolio values is calculated.

- Component VaR for each instrument is calculated as follows:

$$\text{Component VaR} = [(\text{Covariance}) / (\text{Variance})] * \text{Portfolio VaR}$$

2.3 Monte Carlo Simulation Method

Simulation of Risk Factor

For the purpose of Monte Carlo Simulation, risk factors are simulated using stochastic processes such as Geometric Brownian Motion (for Equity and Commodities), Garman-Kohlhagen Process (for Currency) and Hull-White Process or Black Process or Ornstein Uhlen Beck Process or Ho-Lee Process (for Interest Rate). The parameters of each of the stochastic process are available as a download.

Risk factor Scenario under Monte Carlo simulation is derived using stochastic process for each risk factor, Correlation Matrix, and seed value to generate Pseudo random Number. Monte Carlo Simulation Method is three step process:

Step 1

Pseudo Random number is given as input to stochastic process to arrive at simulated risk factor values.

Pseudo Random number is generated using Seed value provided by user. User can provide seed value while defining VaR model, if seed value is not provided by user then system generates random Seed value.

Step 2

Generated Pseudo random number is used to simulate value of each Risk factor using stochastic process assigned to them. In addition to pseudo random number each stochastic process requires there specific inputs like mean and variance. Details of stochastic process for each risk factor are explained below.

Equity and Commodity Risk Factors

Custom equities and benchmark indices and commodities are modeled using Geometric Brownian Motion. The output of this process is the simulated prices of all equities in the portfolio.

Geometric Brownian Motion generates scenario for respective risk factor using Pseudo Random number and three inputs, namely initial value, Mue (mean) and Sigma (Variance) of risk factor.

Geometric Brownian Motion is governed by following equation:

$$dS(t, S) = \mu S dt + \sigma S dW_t.$$

The risk factor for equities is either the benchmark stock index to which it is mapped or the equity itself. Custom equity risk factors are modeled using a Geometric Brownian Motion. In case of equities mapped to the stock index, the index itself is modeled and the index returns are estimated. The price of a particular equity is estimated from the modeled returns on the respective stock index. OFS Market Risk estimates the equity prices using the Capital Asset Pricing Model as follows:

$$E_{t+1} = E_t * \exp(\beta_{im} * R_m)$$

Where,

E_{t+1}	:	Equity Price at time t+1
E_t	:	Equity Price at time t
β_{im}	:	Beta or the sensitivity of asset returns to stock index returns
R_m	:	Return on stock index

Currency Risk Factor

Currencies are modeled using Garman Kohlagen Process. The output of this process is the simulated prices of Exchange rates. Garman Kohlagen Process generates scenario for respective risk factor using four inputs, namely spot value between currencies, Risk Free term structure of first currency, Risk free term structure of second currency and volatility term structure. Initial value provided to equation is current spot value; risk factor scenarios are created by propagating initial value and Garman Kohlagen Process equation preserves the volatility of generated scenarios.

Garman Kohlagen Process is governed by following equation:

$$dS(t, S) = \left(r(t) - r_f(t) - \frac{\sigma(t, S)^2}{2} \right) dt + \sigma dW_t$$

Interest Rates Risk Factor

Interest rates are modeled using the interest rate model specified by the user (Hull White Process, Black Process, Ornstein Uhlen Beck Process or Ho-Lee Process). In case spread over Sovereign yield curve is specified for any interest rate then that spread is applied to the simulated values of Sovereign yield curve to arrive at the simulated values for the given rating. Benchmark indices such as Libor are simulated using the interest rate model specified for Sovereign yield curve.

Hull White Process generates scenario for respective risk factor using Pseudo Random number and three inputs, namely Yield term structure, Mean reversion rate and Sigma (variance). To estimate the Mean Reversion Rate and Speed, each interest rate risk factor's change of rate are computed for the specified number of days. Change of rate is regressed with rate using linear regression. Speed of the mean reversion is $-1 \times \text{slope}$. Mean reversion rate and speed are calculated using NAG library's liner equation "nag_simple_linear_regression (g02cac)". Details description of NAG equation is available on

http://www.nag.com/numeric/CL/nagdoc_cl23/html/G02/g02cac.html.

This process is repeated for all interest rate risk factors. Hull White Process is governed by following equation:

$$dr_t = (\theta(t) - ar_t)dt + \sigma dW_t$$

Black Process generates scenario for respective risk factor using Overnight rate and Volatility term structure. Black Process is governed by following equation:

$$dS_t = \left(r(t) - q(t) - \frac{\sigma(t, S)^2}{2} \right) dt + \sigma dW_t$$

Ornstein Uhlen Beck Process generates scenario for respective risk factor using Pseudo Random number and Overnight rate and speed value as provided by user while defining VaR Model.

Ornstein Uhlen Beck Process is governed by following equation:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$$

Ho-Lee Process generates scenario for respective risk factor using Mean reversion rate and Sigma (variance). Ho-Lee model is equivalent to Hull-White model with a mean-reversion rate as zero.

Interest rate model specified are short-rate models and therefore model the (spot) overnight rate for all Interest Rate Asset Classes. For other maturity buckets the interest rate is modeled using the correlation of the overnight rate with the maturity bucket rate. This is done by calculating the beta coefficient. An Error value is also included in Beta function, error value is with respect to maximum deviation observed in target maturity bucket.

Error value in beta function is computed using iterative process. This error value is to address variance of spot with respect to target time vertex. In certain scenarios where spot rate are stable and target time vertex rate are volatile in nature, beta function's resultant scenario of target time vertex represents the variation as in spot rate and not target time vertex rate. Error value in beta function tends to incorporate the variation of target time vertex in the generated scenario.

Maximum 1000 iterations will be performed to compute error value. Initial Error value is set to 0.5. After each iteration difference between maximum deviation in historical data is compared with maximum deviation generated in simulation. Iteration where above mentioned difference is minimum will be stamped as final scenario set for corresponding maturity time bucket.

$$\beta_{0,i} = \rho_{0,i} * \left(\frac{\sigma_i}{\sigma_0}\right) * \epsilon$$

Where,

$$\begin{aligned} \beta_{0,i} &= \text{Beta Coefficient for } i^{\text{th}} \text{ maturity time bucket} \\ \rho_{0,i} &= \text{Correlation between overnight rate and } i^{\text{th}} \text{ maturity time bucket} \\ \sigma_0 &= \text{Volatility of the overnight rate} \\ \sigma_i &= \text{Volatility of } i^{\text{th}} \text{ maturity time bucket} \\ \epsilon &= \text{Error value} \end{aligned}$$

The log change in the spot rate is multiplied by the Beta coefficient to arrive at the log change to the target maturity time bucket rate with respect to all scenarios.

$$\ln(r_{i,X\text{scenario}}) - \ln(r_{i,\text{spot}}) = \beta_{0,i} [\ln(r_{0,X\text{scenario}}) - \ln(r_{0,\text{spot}})]$$

Where,

$$\begin{aligned} r_{i,X\text{scenario}} &= X^{\text{th}} \text{ scenario rate of } i^{\text{th}} \text{ maturity time bucket} \\ r_{i,\text{spot}} &= \text{Spot rate of } i^{\text{th}} \text{ maturity time bucket} \\ r_{0,X\text{scenario}} &= X^{\text{th}} \text{ scenario rate of overnight time bucket} \\ r_{0,\text{spot}} &= \text{Spot rate of overnight time bucket} \\ \beta_{0,i} &= \text{Beta Coefficient for } i^{\text{th}} \text{ maturity time bucket} \end{aligned}$$

The simulated value of i^{th} -day interest is calculated by solving for $r_{i,X\text{scenario}}$ in the above equation.

While simulating interest rates, the resulting values may be negative. A floor of 0.001 is applied to all the simulated values. Any interest rate going below this value is taken as 0.001 for the purpose of calculations.

CDS Spreads

In case of Credit Defaults Swaps and Collateralized debt Obligations the risk factor to be simulated is the CDS spread. Only the spreads at the shortest maturity are modeled using Geometric Brownian Motion process. The spreads of all other maturities are simulated in a manner similar to β simulation. The beta co-efficient is calculated based on the equation given above. The absolute change in the spread value at the shortest maturity is multiplied with the by the Beta coefficient to arrive at the value of the other maturities.

$$S_{1,i} - S_{1,0} = \beta_{0.5,1} (S_{0.5,i} - S_{0.5,0})$$

Where,

$$\begin{aligned}
 S1,i &= \text{Scenario value of 1 – year spread rate} \\
 S1,0 &= \text{Scenario value of 1 – year spread rate} \\
 S0.5,i &= \text{Scenario value of 6 – month spread rate} \\
 S0.5,0 &= \text{Scenario value of 6 – month spread rate} \\
 B0.5,1 &= \text{Beta Coefficient}
 \end{aligned}$$

Under Historical Simulation, the absolute change in the value of spread rates is taken as the return which is applied to the current spread rate. The formula for calculating spread returns is as follows:

$$R_t = S_t - S_{t-1}$$

Where,

$$\begin{aligned}
 R_t &= \text{Return on spread rates at time } t \\
 S_t &= \text{Spread rate at time } t \\
 S_{t-1} &= \text{Spread rate at time } t - 1
 \end{aligned}$$

Step 3

Correlated Risk factor scenarios are then derived using Correlation Matrix and simulated risk factor generated in above step.

Simulation of Portfolio Value

Simulated portfolio value is the price of the portfolio under each scenario.

The instrument value for all instrument as part of given portfolio is calculated under all simulated scenario. The simulated portfolio value in the reporting currency is derived as sum of all instrument prices under each scenario.

P&L Distribution

The P&L Distribution is Profit or Loss of the portfolio under each scenario against the portfolio value as on the current day.

Portfolio VaR and CVaR

The VaR and CVaR values obtained from the P&L Distribution are based on the confidence level and horizon specified in the Market Risk VaR Model.

Based on the confidence level “ α ” the 1-day α -VaR is the $(1 - \alpha)$ th value in the arranged P&L values. The confidence level “ α ” must be entered in OFS Market Risk application. The t-day α -VaR is estimated by multiplying the 1-day α -VaR by \sqrt{t} .

CVaR is calculated as the Expected Value of all simulated data points that exceed the VaR. It is the expected loss conditional on the occurrence of loss greater than VaR. The P&L distribution is calculated as follows:

$$\text{CVaR} = E(Y / Y > \text{VaR})$$

Where,

Y : Value of Simulated Loss

Thus, it is the arithmetic average of all simulation points exceeding the VaR value.

Component VaR

Under the Simulation Methods to risk measure estimation, both Monte Carlo as well as Historical, Component VaR is calculated as the contribution of each position to the Portfolio VaR as follows:

- The scenario position values are estimated for “n” instruments in the portfolio for “m” scenarios of prices and converted to the home currency.
- The variance of the scenario portfolio values is estimated as follows:

$$\text{Variance} = 1/m \sum_{j=1}^m (jP_{t+1} - \mu_j)^2$$

Where,

m : Number of scenario portfolio values

jP_{t+1} : Scenario portfolio value

μ_j : Mean of the scenario portfolio values

- The covariance between the scenario position values of each instrument and the scenario portfolio values is estimated as follows:

$$\text{Covariance} = 1/m \sum_{k=1}^m \sum_{j=1}^m (x_k - \mu_k) (jP_{t+1} - \mu_j)$$

Where,

x_k : Scenario position values of instrument “i”

The covariance of each position with the scenario portfolio values is calculated.

- Component VaR for each instrument is calculated as follows:

$$\text{Component VaR} = [(\text{Covariance}) / (\text{Variance})] * \text{Portfolio VaR}$$

2.4 Back Testing

Back testing refers to the procedure of verifying the accuracy of the VaR model by comparing the realized P&L value with the value predicted by the VaR model.

OFS Market Risk back tests VaR estimates using Hypothetical P&L. Hypothetical P&L is defined as the gain / loss from a position at some specific time (Example, end of local business day) resulting from changes in market over the next twenty four hours, assuming a static portfolio

Actual P&L is affected by internal risk management policy and practice. Here a dynamic portfolio is assumed. OFS Market Risk back tests VaR estimates using the following methods:

- Simple Back Test
- Kupiec Test

A simple back test involves counting the number of loss exceptions for a given time period. Kupiec Test counts the number of exceptions in any given sample and performs a Likelihood-Ratio-Test at a given confidence level. Back testing is done for the days.

Back Testing comprises of calculation of the following:

- Hypothetical P&L
- Profit/Loss Exceptions
- P-value
- Critical Value
- Loss Exception Deviation
- Average Loss Duration
- Loss Duration Deviation

Calculation of Hypothetical P&L

OFS Market Risk calculates the hypothetical P&L of the portfolio assuming that the portfolio composition remains unchanged over the VaR horizon. Portfolio value at time “t” is calculated based on the actual prices of the instruments as on time “t” and the position held in the instruments. Hypothetical portfolio value at time “t+1” is calculated based on the actual prices of the instruments as on time “t+1” and the position held in the instruments at time “t”. Hypothetical P&L is calculated as the difference between the hypothetical portfolio value as of time “t+1” and the portfolio value as of time “t”.

The formula for computing hypothetical portfolio value is as follows:

$$HP_{t+1} = \sum_{i=1}^n [w_t(i) p_{t+1}(i)]$$

Where,

- HP_{t+1} : Hypothetical Value of Portfolio at “t+1”
- w_t(i) : Number of units of instrument “i” in the portfolio at time “t”
- p_{t+1}(i) : Price of instrument “i” at time “t+1”

Calculation of Profit/Loss Exceptions

OFS Market Risk compares the actual as well as the hypothetical portfolio loss with the estimated VaR value for the back testing period specified by the user in order to find the number of

exceptions. Loss exception is the number of times the actual loss exceeds the estimated VaR value for a given confidence level. OFS Market Risk also calculates the profit exceptions.

Calculation of P-value

P-value refers to the probability of the frequency of exceptions for a given level of confidence.

P-Value is calculated by passing the number of back testing points, number of Actual exceptions and the exception probability to the Binomial Distribution and then passing the probability calculated to Chi square distribution.

Where,

Number of back testing points is number of back testing days taken as input from user in VaR Model definition

Number of Actual exceptions is Profit/Loss Exceptions”

Exception probability is number of Actual Exceptions / number of back testing points

OFS Market Risk is using following Binomial Distribution equation:

Let X denote a random variable having a Binomial distribution with parameters n and p ($n \geq 0$ and $0 < p < 1$).

Then,

$$Prob\{X = k\} = \binom{n}{k} p^k (1 - p)^{n-k}, \text{ for } k = 0, 1, \dots, n.$$

Where

n = number of back testing points

p = exception probability

k = number of Actual exceptions

OFS Market Risk is using following Chi-Distribution equation:

The lower tail probability for the χ^2 distribution with ν degrees of freedom, for calculating probability $P(X \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a χ^2 distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

Where,

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_{0.0}^x X^{(\nu/2)-1} e^{-X/2} dX, x \geq 0, \nu > 0$$

Estimation of Critical Value

Critical value is asymptotically Chi Square distributed with one degree of freedom. OFS Market Risk compares the P-value with the critical value at 5% and 1% significance level, on both the loss as well as profit tail of the P&L distribution, in order to check the validity of the Model.

Critical Value is calculated as two step process. First step is to calculate P-value by passing the number of back testing points, number of Actual exceptions and the probability as 1% (for Critical value at 1%) and 5% (for critical value at 5%) to the Binomial Distribution. Resultant output is further processed using Chi-Distribution to generate probability for the χ^2 distribution, to arrive at Critical Value.

Calculation of Loss Exception Deviation

Loss Exception Deviation is the absolute average deviation of the loss exceptions from the CVaR. It is calculated as follows:

$$\text{Loss Exception Deviation} = \frac{\sum_{t=1}^n (L_t - CVaR_t)}{n}$$

Where,

n = Number of loss exceptions

Lt = Loss exceeding VaR at time “t”

CVaRt = Conditional VaR of the portfolio at time “t”

Calculation of Average Loss Duration

Average Loss Duration is the average time interval between successive loss exceptions.

Calculation of Loss Duration Deviation

Loss Duration Deviation refers to the standard deviation of time interval between successive loss exceptions. The standard deviation of time interval between successive loss exceptions is calculated.

It is calculated as per the formula given below:

$$\text{Loss Duration Deviation} = \left(\frac{\sum_{t=1}^n (\text{Average Deviaton Loss} - \text{Deviatton Loss})}{n} \right)^{1/2}$$

Where,

$$\text{Average Deviation Loss} = \left(\frac{\sum_{t=1}^n (\text{Deviatton Loss})}{n} \right)$$

n= Number of loss exceptions

2.5 Stress Testing

The Stress Testing Framework is integrated with the Market Risk solution to deliver enhanced stress testing capability. Stress testing framework allows the users to define variables, specify variable as well as rule based shocks and combine them into stress scenarios.

Shocks can be defined on the following variables as part of market risk:

- Volatility
- Correlation
- Risk factors

Volatility

In case of volatility shocks, shocks are specified on the volatility of one or multiple risk factor returns. The stressed volatilities are used in the Market Risk VaR Model Run in order to estimate the risk measures. Volatility-Correlation Run is not re-executed in this case.

Correlation

In case of correlation shocks, shocks are specified on the correlation between 2 risk factors. The stressed correlation data is used in the Market Risk VaR Model Run in order to estimate the risk measures. Volatility-Correlation Run is not re-executed in this case.

Risk Factors

Shocks specified on one or multiple risk factors such as equity prices, exchange rates and so on which comprise of risk factor shocks. Risk factor shocks can be applied in 3 ways.

- **Shocks applied to Volatility/Correlation Run**

This scenario affects both the Volatility – Correlation Run as well as the Market Risk VaR Model Run. In this case, the historical data of risk factors is replaced with the stressed values of risk factors. The Volatility-Correlation Run is re-executed in order to estimate the stressed volatility-correlation matrix. This stressed volatility-correlation data is used in the Market Risk VaR Model Run for estimating risk measures.

- **Shocks applied to Market Risk VaR Model Run**

In this case, the risk factor values are replaced with the stressed values based on the shocks applied. The stressed risk factor values are used for pricing instruments and for simulating the risk factor returns in the Market Risk VaR Model Run in order to estimate the risk measures. Volatility-Correlation Run is not re-executed in this case. The volatility-correlation data estimated previously is re-used in the Market Risk VaR Model Run.

- **Shocks applied to Volatility/Correlation Run and Market Risk VaR Model Run**

This scenario affects both the Volatility – Correlation Run as well as the Market Risk VaR Model Run. In this case, the historical data of risk factors is replaced with the stressed values of risk factors. The Volatility-Correlation Run is re-executed in order to estimate the stressed volatility-correlation matrix. This stressed volatility-correlation data is used in the Market Risk

VaR Model Run for estimating risk measures. In addition, the stressed values of risk factors are used for pricing instruments and simulating risk factor returns.



Underlying Methodology of the Models in OFS
Market Risk

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